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2021.10

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Maison des Sciences Économiques, 106-112 boulevard de L'Hôpital, 75647 Paris Cedex 13
https://centredeconomiesorbonne.cnrs.fr/
ISSN : 1955-611X

Dropping the Cass Trick and Extending Cass' Theorem to Asymmetric Information and Restricted Participation

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(March 2021)

Abstract

In a celebrated 1984 paper, David Cass provided an existence theorem for financial equilibria in incomplete markets with exogenous yields. The theorem showed that, when agents had symmetric information and ordered preferences, equilibria existed on purely financial markets, supported by any collection of state prices. This theorem built on the so-called "Cass trick", along which one agent had an Arrow-Debreu budget set, with one single constraint, while the other agents were constrained a la Radner (1972), that is, in every state of nature. The current paper extends Cass' theorem to asymmetric information, non-ordered preferences and restricted participation. It refines De Boisdeffre (2007), which characterized the existence of equilibria with asymmetric information by the no-arbitrage condition on purely financial markets. The paper defines no arbitrage prices with asymmetric information. It shows that any collection of state prices, in the agents' commonly expected states, supports an equilibrium. This result is proved without using the Cass trick, in the sense that budget sets are defined symmetrically across all agents. Thus, the paper suggests, in the symmetric information case, an alternative proof to Cass'.

Key words: sequential equilibrium, perfect foresight, existence of equilibrium, rational expectations, incomplete markets, asymmetric information, arbitrage.

JEL Classification: D52

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1 Introduction

The current contribution needs be introduced with reference to standard concepts and properties of sequential equilibria, with symmetric and asymmetric information.

With symmetric information, the classical definition of sequential equilibrium relies on the assumption that agents know the map between future random states and the price to prevail on every spot market. Under this so-called '*perfect foresight*' or '*rational expectation*' hypothesis, the existence of equilibrium has been extensively studied since Radner (1972). With nominal and numéraire assets, the span of payoffs does not depend on prices. This insures the full existence of equilibrium in standard conditions, as shown by Cass (1984, 2006), Duffie (1987), Werner (1985), for nominal assets, and Geanakoplos-Polemarchakis (1986), for numéraire assets.

The literature shows an essential real indeterminacy of equilibrium with nominal assets (Balasko-Cass, 1989; Geanakoplos-Mas-Colell, 1989), whereas no-arbitrage prices coincide with equilibrium asset prices (Cass, 1984). Contrarily, with numéraire assets, Geanakoplos-Polemarchakis (1986) proves the generic local uniqueness of equilibrium, but does not extend Cass' result. The extension of Cass (1984) to numéraire assets and asymmetric information is poposed in De Boisdeffre (2021, a).

With other types of assets, the existence of equilibrium is not guaranteed, as shown by Hart (1975), when agents forecast prices perfectly. Hart's counterexample is based on the collapse of the span of assets' payoffs, that occurs exceptionally at clearing-market prices. One response to that problem is to show, along Duffie-Shafer (1985), that the fall in rank of endogenous yields is, indeed, exceptional, and equilibrium therefore exists generically in payoffs and endowments. Geanakoplos-

Shafer (1990), Bich-Cornet (2004, 2009), in the symmetric information case, and De Boisdeffre (2021, a), in the asymmetric information case, resume this argument.

Another approach is to drop the perfect foresight hypothesis, and let, instead, agents have an '*endogenous uncertainty*' over future spot prices, akin to Kurz' (1994). Under a milder '*correct foresight*' assumption, the full existence of sequential equilibrium may be restored for all types of assets and private information signals (see De Boisdeffre, 2021, b). For expositional purposes, the current paper does not retain this approach, which requires to deal with infinite dimensional economies.

When agents have asymmetric or incomplete information, they seek to learn more information from markets. The traditional inference mechanism is described by the REE (rational expectations equilibrium) model by assuming, quoting Radner (1979), that "*agents have a 'model' or 'expectations' of how equilibrium prices are determined*". Under this assumption, agents know the maps between private information signals and equilibrium prices, along a so-called '*forecast function*'. Since equilibrium prices are typically distinct across agents' joint information signals, that forecast function would theoretically enable consumers to infer all information detained by the other agents from observing such '*separating*' prices. However, that function depends on all consumers' characteristics, which are private. Inferring and using it to reach a REE is seen as theoretical, if realistic. Along De Boisdeffre (2016), markets would not reveal information via a price model, but through arbitrage.

With this approach, in the simplest setting to study arbitrage, Cornet-De Boisdeffre (2002) suggests an alternative model to Radner's (1979), where asymmetric information is represented by private signals, informing each agent that tomorrow's true state will be in a subset of the state space. This specification reflects the fact than an information may always reduce a set of possibilities. The latter paper generalizes the definitions of equilibrium, no-arbitrage prices and no-arbitrage condition to asymmetric information. From De Boisdeffre (2007, 2016), the latter no-arbitrage condition can be reached by agents observing available transfers, and characterizes the existence of equilibrium, on purely financial markets. Such results differ from Radner's (1979) inferences and the ensuing generic existence of a REE.

If De Boisdeffre (2007) proves the full existence of equilibrium, it does not assess whether the two concepts of no-arbitrage price and equilibrium asset price coincide, as in the symmetric information economy studied by Cass (1984, 2006). To prove this equivalence, the author relies on the so-called '*Cass trick*', which lets one agent have an Arrow-Debreu budget set, with one single constraint, and the other agents be constrained a la Radner (1972), i.e., in each state of nature. Without using the Cass trick, the current paper extends Cass' theorem to asymmetric information, non-ordered preferences and restricted portfolio participation. It defines no-arbitrage prices with reference to individual state prices. It shows that any collection of state prices, in the agents' commonly expected states, supports an equilibrium. In the particular setting of Cass, it thus suggests an alternative proof.

The current paper is not the first generalization of Cass' theorem which drops the Cass trick (see Cornet-Gopalan, 2010). But it is, to our best knowledge, the first, after De Boisdeffre (2021, a), to extend the theorem to asymmetric information. The latter paper extends Cass' (1984) result to nominal and numéraire asset markets under asymmetric information. But it uses differential topology arguments, which only yield *interior* equilibrium allocations. Moreover, the latter paper still applies the Cass trick, and provides no other insight on the price equivalence, stated above.

The current paper aims to fill these gaps. It also addresses restricted portolio participation. This extension of the Cass model has been extensively studied, in the symmetric information case, by several papers culminating with Cornet-Gopalan (2010). Differently from Cornet-Gopalan, we chose not to go beyond two periods, for expositional purposes. The extension of a purely financial economy from two periods to multiple periods poses no conceptual difficulty and is standard. But it implies introducing additional nodes and notations, which, in the present case, would have hampered the simplicity and focus of the model. This extension is all but necessary to introduce asymmetric information in the model and is deferred.

The paper is organized as follows: Section 2 presents the model. Section 3 states and proves the existence theorem in the general model, and illustrates the theorem when one agent has full information. An Appendix proves technical Lemmas.

2 The model

We consider a pure-exchange financial economy with two periods, $t \in \{0,1\}$, and an uncertainty, at $t = 0$, upon which state of nature will randomly prevail at $t = 1$. The economy is finite in the sense that the sets, I, S, L and J, respectively, of consumers, states of nature, consumption goods and assets are all finite. The observed state at $t = 0$ is denoted by $s = 0$ and we let $\Sigma' := \{0\} \cup \Sigma$, whenever $\Sigma \subset S$.

2.1 Markets and information

Agents consume or exchange the consumption goods, $l \in L$, on both periods' spot markets. At $t = 0$, each agent, $i \in I$, receives privately the correct information that tomorrow's true state will be in a subset, S_i, of S. We assume costlessly that $S = \cup_{i \in I} S_i$. Thus, the pooled information set, $\underline{\mathbf{S}} := \cap_{i \in I} S_i$, contains the true state, and the relation $\underline{\mathbf{S}} = S$ characterizes symmetric information.

We let $P := \{p := (p_s) \in \mathbb{R}^{L\times \underline{\mathbf{S}}'} : \|p_s\| \leqslant 1, \forall s \in \underline{\mathbf{S}}'\}$ be the set of admissible commodity prices, which each agent is assumed to observe, or anticipate perfectly, a la Radner (1972). Moreover, each agent with an incomplete information forms her private forecasts in the unrealizable states she expects. Such forecasts, (s, p^i_s), are pairs of a state, $s \in S_i\backslash\underline{\mathbf{S}}$, and a price, $p^i_s \in \mathbb{R}^L_{++}$, that the generic i^{th} agent believes to be the conditional spot price in state s. Non-fully informed agents may agree or disagree on forecasts, which never obtain and are henceforth given, along De Boisdeffre (2007).

Agents may operate financial transfers across states in S' (actually in $\underline{\mathbf{S}}'$) by exchanging, at $t = 0$, finitely many nominal assets, $j \in J$, which pay off, at $t = 1$, conditionally on the realization of the state. We assume that $\#J \leqslant \#\underline{\mathbf{S}}$, so that financial markets be typically incomplete. Assets' payoffs define a $S \times J$ matrix, V, whose generic row in state $s \in S$, denoted by $V(s) \in \mathbb{R}^J$, does not depend on prices. Thus, at asset price, $q \in \mathbb{R}^J$, agents may buy or sell portfolios of assets, $z = (z_j) \in \mathbb{R}^J$, for $q \cdot z$ units of account at $t = 0$, against the promise of delivery of a flow, $V(s) \cdot z$, of conditional payoffs across states, $s \in S$. Moreover, the exchange of assets may be restricted, e.g., if agents are unaware or have no access to some available transfers. Participation to markets is then said to be restricted. For each $i \in I$, we let $Y_i \subset \mathbb{R}^J$ be the i^{th} agent's set of admissible portfolios, henceforth set as given.

2.2 The consumer's behaviour and concept of equilibrium

Each agent, $i \in I$, receives an endowment, $e_i := (e_{is})$, granting the commodity bundles, $e_{i0} \in \mathbb{R}^L_+$ at $t = 0$, and $e_{is} \in \mathbb{R}^L_+$, in each expected state, $s \in S_i$, if it prevails. Given the market prices, $p := (p_s) \in P$, for goods, $q \in \mathbb{R}^J$, for assets, and her possible forecasts, the generic i^{th} agent's consumption set is $X_i := \mathbb{R}^{L\times S'_i}_+$, and budget set is:

$$B_i(p,q) := \{\ (x,z) \in X_i{\times}Y_i :\ \ p_0{\cdot}(x_0 - e_{i0}) \leqslant -q{\cdot}z \ \ \textit{and}\ \ p_s{\cdot}(x_s - e_{is}) \leqslant V(s){\cdot}z,\ \forall s \in \underline{\mathbf{S}}\ \ \textit{and}$$
$$p^i_s{\cdot}(x_s - e_{is}) \leqslant V(s){\cdot}z,\ \forall s \in S_i\backslash\underline{\mathbf{S}}\ \}.$$

Each consumer, $i \in I$, is endowed with a complete preordering, $\precsim_i$, over her consumption set, representing her preferences. Her strict preferences, $\prec_i$, are represented, for each $x \in X_i$, by the set, $P_i(x) := \{ \ y \in X_i : x \prec_i y \ \}$, of consumptions which are strictly preferred to x. The above economy is denoted by $\mathcal{E} = \{(I, S, L, J), V, (S_i), (p_s^i), (e_i), (\prec_i)\}$ and yields the following concept of equilibrium:

Definition 1 *A collection of prices,* $p = (p_s) \in P$, $q \in \mathbb{R}^J$, *& decisions,* $(x_i, z_i) \in B_i(p, q)$, *for each* $i \in I$, *is an equilibrium of the economy,* $\mathcal{E}$, *if the following conditions hold:*
(a) $\forall i \in I, \ (x_i, z_i) \in B_i(p, q)$ *and* $P_i(x_i) \times \mathbb{R}^J \cap B_i(p, q) = \varnothing$;
(b) $\sum_{i \in I} (x_{is} - e_{is}) = 0, \ \forall s \in \underline{\mathbf{S}}'$;
(c) $\sum_{i \in I} z_i = 0$.

Endowments and preferences are called standard under the Assumptions:

- ***A1*** *(strict monotonicity):* $\forall (i, x, y) \in I \times X_i \times X_i, \ (x \leqslant y, \ x \neq y) \Rightarrow (x \prec_i y)$;
- ***A2*** *(strong survival):* $\forall i \in I, \ e_i \in \mathbb{R}_{++}^{L \times S_i'}$;
- ***A3*** *for every* $i \in I$, $\prec_i$ *is lower semi-continuous, convex-open-valued and such that* $x \prec_i x + \lambda(y - x)$, *whenever* $(x, y, \lambda) \in X_i \times P_i(x) \times \,]0, 1]$.

We now present algebraic notations and properties that will be used throughout.

2.3 The model's notations and payoff structure

For every $(i, s, z) \in I \times S \times \mathbb{R}^J$, we define the following vector spaces and sums:

- $Z_i^o := \{z \in \mathbb{R}^J : V(s) \cdot z = 0, \forall s \in S_i\}$ & orthogonal complement, $Z_i := \sum_{s \in S_i} \mathbb{R}V(s)$;
- $\underline{Z}^o := \{z \in \mathbb{R}^J : V(s) \cdot z = 0, \ \forall s \in \underline{\mathbf{S}}\}$ and orthogonal complement, $\underline{Z} := \sum_{s \in \underline{\mathbf{S}}} \mathbb{R}V(s)$;
- $Z^o := \sum_{s \in S_i} Z_i^o$, its orthogonal complement, $Z := \cap_{i \in I} Z_i$, and $Z^* := Z \cap \underline{Z}^o$;
- $z = z^1 + z^* + z^2$, the orthogonal decomposition of z on $Z^o \oplus Z^* \oplus \underline{Z}$.

We notice that the relation $\underline{Z} \subset Z$ holds for any financial structure, (S_i), and that $Z^* = \{0\}$ holds, if and only if $Z = \underline{Z}$ (in particular, if one agent is fully informed). We denote $j^1 := \dim Z^o$, $j^* := \dim Z^*$ and $j^2 := \dim \underline{Z}$, which satisfy: $\#J = j^1 + j^* + j^2$.

For convenience, we henceforth assume costlessly, by combining and reordering assets if needed, that portfolios selected amongst the first j^1 assets (if $j^1 > 0$) belong to Z^o, those from the subsequent j^* assets (if $j^* > 0$) belong to Z^*, and those from the last j^2 assets (if any) belong to $\underline{Z}$. For each $s \in \underline{\mathbf{S}}$, the relation $V(s) = V(s)^2$ follows from these assumptions, that is, the first j^1+j^* components of $V(s)$ are null.

Remark 1 The above structure of payoffs may always be assumed and obtained by replacing (if needed) the columns of the matrix V by linear combinaisons of these columns, without changing the span of payoffs and, hence, the financial structure.

Financial markets and participation are called standard under the Assumptions:

- ***A4*** *for every* $i \in I$, Y_i *is closed, convex and contains zero*;
- ***A5*** *for every* $i \in I$, $\underline{Z}^o + Y_i \subset Y_i$;
- ***A6*** *(riskless asset)* $\exists z^+ \in \cap_{i \in I} Y_i : V(s) \cdot z^+ > 0,\ \forall s \in \underline{\mathbf{S}}$.

The financial economy, $\mathcal{E}$, is said to be standard if preferences, endowments, financial markets and participation are standard, along Assumptions *A1* to *A6*.

Remark 2 Under symmetric information, the relation $\underline{Z}^o = \{0\}$ holds, from the elimination of redundant assets, and Assumption *A5* vanishes. Assumption *A4* is made throughout Cornet-Gopalan (2010) and is a minimal requirement in most models of portfolio choice with financial constraints (Elsinger and Summer, 2001). Assumption *A6*, which states that agents are always proposed to save a small amount of cash to insure risk in all realizable states, is met on all actual markets.

Under asymmetric information, Assumptions *A5-A6* are made for technical purposes and serve to prove, in particular, Claim 3 below. Assumption *A5* states that what would be a worthless portfolio to an informed agent may always be combined to an admissible portfolio. This possibility may be explained by the role of financial intermediaries, proposing to insure agents against their idiosyncratic risks, represented by states $s \in S \backslash \underline{\mathbf{S}}$. In the current model, the latter states never prevail.

We now present no-arbitrage prices, state prices, and some of their properties.

2.4 No-arbitrage prices and state prices with asymmetric information

We start with a definition:

Definition 2 *A no-arbitrage price is an asset price, $q \in \mathbb{R}^J$, which meets one of the following equivalent conditions, and we let $\mathcal{NA}$ be their set:*

(a) $\nexists (i,z) \in I \times \mathbb{R}^J$, $-q \cdot z \geqslant 0$ and $V(s) \cdot z \geqslant 0$, $\forall s \in S_i$, with one strict inequality;

(b) $\forall i \in I$, $\exists \lambda_i := (\lambda_{is}) \in \mathbb{R}^{S_i}_{++}$, $q = \sum_{s \in S_i} \lambda_{is} V(s)$.

Scalars, $(\lambda_{is}) \in \times_{i \in I} \mathbb{R}^{S_i}_{++}$, which meet the above condition (b), are said to support the no-arbitrage price, $q \in \mathcal{NA}$, and called (individual) state prices. For every $\lambda := (\lambda_s) \in \mathbb{R}^{\underline{\mathbf{S}}}_{++}$, we denote $\mathcal{NA}(\lambda) := \{q \in \mathcal{NA} : q^2 = \sum_{s \in \underline{\mathbf{S}}} \lambda_s V(s)\}$ and let $\mathcal{NAC} = \cup_{\lambda \in \mathbb{R}^{\underline{\mathbf{S}}}_{++}} \mathcal{NA}(\lambda)$.

N.B. The equivalence between Conditions (a) and (b) of Definition 2 is standard and proved in Cornet-De Boisdeffre (2002, Lemma 1, p. 398).

We henceforth assume that (S_i) is arbitrage-free, i.e., admits a no-arbitrage price.

<u>*Remark 3*</u> The information structure, (S_i), is arbitrage-free when it is symmetric. Otherwise, from Assumption *A5*, it is costless to assume that (S_i) is arbitrage-free. Indeed, it follows from Cornet-De Boisdeffre (2009) that agents may narrow down

their information sets, if needed, from observing informative portfolios, which all belong to $\underline{Z}^o$. Then, agents infer these sets' coarsest arbitrage-free refinements.

The following heuristic example and Claim 1 show that the sets $\mathcal{NA}$ and $\mathcal{NAC}$ differ, in general, but that any collection of state prices, $\lambda := (\lambda_s) \in \mathbb{R}^{\underline{\mathbf{S}}}_{++}$, supports a no-arbitrage price, $q \in \mathcal{NA}(\lambda)$, and supports an equilibrium, from Theorem 1 below.

<u>*Example*</u> Consider an economy, $\mathcal{E}$, with two agents, $i \in \{1,2\}$, three states, $s \in \{1,2,3\}$, an information stucture, $S_1 := \{1,2\}$ and $S_2 := \{2,3\}$, and one asset, whose price is $q = 1$. For a payoff matrix $V = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, the relations $Z^* = \mathbb{R}$, $\underline{Z} = \{0\}$, $q \in \mathcal{NAC}$ hold. For $V = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, the relations $Z^* \subset \underline{Z}^o = \{0\}$ and $q \in \mathcal{NA} \not\subseteq \mathcal{NAC} = -\mathbb{R}_{++}$ hold.

Claim 1 *The following Assertions hold:*

(i) $\forall \lambda \in \mathbb{R}^{\underline{\mathbf{S}}}_{++}$, $\mathcal{NA}(\lambda) \neq \varnothing$*;*

(ii) $\mathcal{NAC} \subset \mathcal{NA}$*, but* $\mathcal{NA} \not\subseteq \mathcal{NAC}$*, in general.*

Proof Assertion (i) Let $\lambda := (\lambda_s) \in \mathbb{R}^{\underline{\mathbf{S}}}_{++}$ be given. As assumed above, $\mathcal{NA} \neq \varnothing$. Hence, we let $\overline{q} \in \mathcal{NA}$ and $(\lambda_{is}) \in \times_{i \in I} \mathbb{R}^{S_i}_{++}$ be given, such that $\overline{q} = \sum_{s \in S_i} \lambda_{is} V(s)$, for each $i \in I$. Since $\overline{q}$ is a no-arbitrage price, the relation $\overline{q}^1 = 0$ holds ($\overline{q} \cdot z = 0$ if $z \in Z^o$). We refer to the notations, definitions and characteristics of the assets, stated in sub-Section 2.3. We recall that $V(s) = V(s)^2$ may be assumed for every $s \in \underline{\mathbf{S}}$. For each $i \in I$, we define $q_i \in \underline{Z}$ by $q_i = \sum_{s \in S_i \setminus \underline{\mathbf{S}}} \lambda_{is} V(s)^2$, if $S_i \neq \underline{\mathbf{S}}$, and $q_i = 0$ otherwise.

Since $q_i \in \underline{Z}$, there exists a vector $(\mu_{is}) \in \mathbb{R}^{\underline{\mathbf{S}}}$, such that $q_i = \sum_{s \in \underline{\mathbf{S}}} \mu_{is} \, V(s)$, for every $i \in I$. For $N \in \mathbb{N}$ large enough, the relations $|\frac{\mu_{is}}{N}| < \lambda_s$ hold, for every pair

$(i,s) \in I \times \underline{\mathbf{S}}$. Then, we let $\gamma_i := (\gamma_{is}) \in \mathbb{R}^{S_i}_{++}$ be defined, for each $i \in I$, by $\gamma_{is} = \frac{\lambda_{is}}{N}$, for every $s \in S_i \backslash \underline{\mathbf{S}}$, and $\gamma_{is} = \lambda_s - \frac{\mu_{is}}{N}$, for every $s \in \underline{\mathbf{S}}$. By construction, the state prices, $\gamma_i := (\gamma_{is}) \in \mathbb{R}^{S_i}_{++}$, defined for all $i \in I$, support some no-arbitrage price, $q \in \mathcal{NA}(\lambda)$. □

Assertion (ii) results from the definitions and the heuristic example above. □

3 The existence theorem

A standard economy, $\mathcal{E}$, and vectors, $\lambda := (\lambda_s) \in \mathbb{R}^{\underline{\mathbf{S}}}_{++}$ and $\bar{q} = \sum_{s \in \underline{\mathbf{S}}} \lambda_s V(s)$ are henceforth given and we prove that the state prices, λ, support an equilibrium:

Theorem 1 *Let $\lambda \in \mathbb{R}^{\underline{\mathbf{S}}}_{++}$ be given. A standard economy, $\mathcal{E}$, admits an equilibrium, $(p, q, [(x_i, z_i]) \in P \times \mathcal{NA}(\lambda) \times (\times_{i \in I} B_i(p,q))$ along Definitions 1 and 2 above.*

The proof's main argument is the Gale-Mas-Colell (1975, 1979) fixed-point-like theorem. We apply this theorem to lower semi-continuous reaction correspondences, which are defined on convex compact sets and formally represent agents' behaviours. In particular, in the symmetric information case, the proof is an alternative to Cass'.

Sub-Section 3.1 derives from the economy $\mathcal{E}$ an auxiliary compact economy. Sub-Section 3.2 defines the reaction correspondences in the compact economy, to apply the GMC theorem. A so-called (with slight abuse) "*fixed point*" obtains. Sub-Section 3.3 derives from this fixed point an equilibrium of the economy $\mathcal{E}$. Sub-Section 3.4 illustrates the result of Theorem 1 when one agent is fully informed.

3.1 An auxiliary compact economy with modified budget sets

Using the notations and assumptions of sub-Section 2.3, we define the price set, $Q := \{q \in Z^* : \|q\| \leqslant 1\}$. Under Assumptions *A4-A5*, the set $Z_i \cap Y_i$ is closed, convex and contains zero, for each $i \in I$, and coincides with the orthogonal projection of Y_i on Z_i. The following sets are, therefore, well defined, for every $(i,p,q) \in I \times P \times Q$:

$$B_i^1(p,q) := \{\ (x,z) \in X_i \times (Z_i \cap Y_i) :\ p_0 \cdot (x_0 - e_{i0}) + q \cdot z + \textstyle\sum_{s \in \underline{\mathbf{S}}} \lambda_s p_s \cdot (x_s - e_{is}) \leqslant 1,$$
$$p_s \cdot (x_s - e_{is}) \leqslant V(s) \cdot z,\ \forall s \in \underline{\mathbf{S}},\ \ and\ \ p_s^i \cdot (x_s - e_{is}) \leqslant V(s) \cdot z,\ \forall s \in S_i \backslash \underline{\mathbf{S}}\ \};$$
$$\mathcal{A}(p,q) := \{[(x_i,z_i)] \in \times_{i \in I} B_i(p, q+\overline{q}) :\ \textstyle\sum_{i \in I} (x_{is} - e_{is})_{s \in \underline{\mathbf{S}}'} = 0,\ \ (z_i) \in \times_{i \in I} Z_i,\ \ \sum_{i \in I}\ z_i \in Z^o\}.$$

Lemma 1 $\exists r > 0 : \forall (p,q) \in P \times Q,\ \forall\ [(x_i,z_i)] \in \mathcal{A}(p,q),\ \sum_{i \in I} (\|x_i\| + \|z_i\|)\ < r$

Proof : See the Appendix. □

Along Lemma 1, we let $X_i^* := \{x \in X_i : \|x\| \leqslant r\}$ and $Z_i^* := \{z \in Z_i \cap Y_i : \|z\| \leqslant r\}$, and define, for every $(i, p := (p_s), q) \in I \times P \times Q$, the following convex compact sets:

$$B_i'(p,q) := \{\ (x,z) \in X_i^* \times Z_i^* : p_0 \cdot (x_0 - e_{i0}) + q \cdot z + \textstyle\sum_{s \in \underline{\mathbf{S}}} \lambda_s p_s \cdot (x_s - e_{is}) \leqslant \gamma_{(p,q)},$$
$$p_s \cdot (x_s - e_{is}) \leqslant V(s) \cdot z,\ \forall s \in \underline{\mathbf{S}},\ \ and\ \ p_s^i \cdot (x_s - e_{is}) \leqslant V(s) \cdot z,\ \forall s \in S_i \backslash \underline{\mathbf{S}}\ \},$$

where $\gamma_{(p,q)} := 1 - \min(1, \|p\| + \|q\|)$, so that $B_i'(p,q) \subset B_i^1(p,q)$.

Claim 2 *For every $i \in I$, B_i' is upper semicontinuous.*

Proof Let $i \in I$ be given. The correspondences B_i' is, as standard, upper semicontinuous, for having a closed graph in a compact set. □

3.2 The fixed-point-like argument

Budget sets were modified in sub-section 3.1, so that their interiors be non-empty. This was required to prove the lower semi-continuity of the reaction correspondences of Lemma 2, below. For every $(p,q) \in P \times Q$, these interior budget sets are as follows:

$$B_i''(p,q) := \{\ (x,z) \in X_i^* \times Z_i^* : p_0 \cdot (x_0 - e_{i0}) + q \cdot z + \textstyle\sum_{s \in \underline{\mathbf{S}}} \lambda_s p_s \cdot (x_s - e_{is}) < \gamma_{(p,q)}\ \ and$$
$$p_s \cdot (x_s - e_{is}) < V(s) \cdot z,\ \forall s \in \underline{\mathbf{S}},\ \ and\ \ p_s^i \cdot (x_s - e_{is}) < V(s) \cdot z,\ \forall s \in S_i \backslash \underline{\mathbf{S}}\ \},$$ for each $i \in I$.

Claim 3 *The following Assertions hold, for each* $i \in I$*:*

(i) $\forall (p,q) \in P \times Q,\ B''_i(p,q) \neq \varnothing$*;*

(ii) *the correspondence* B''_i *is lower semicontinuous.*

Proof Let $i \in I$, $z^+ \in Q^\perp$, along Assumptions *A6*, and $(p,q) \in P \times Q$ be given.

Assertion (i) If $\gamma_{(p,q)} = 1$, from Assumptions *A4-A6*, the relation $(0, t.z^+) \in B''_i(p,q)$ holds, for $t > 0$ small enough. If $p \neq 0$, from Assumptions *A2-A4-A6* and the relation $z^+ \in Q^\perp$, there exist $(x,t) \in X_i \times \mathbb{R}_{++}$, such that $(x, t.z^+) \in B''_i(p,q)$. If $q \neq 0$, from Assumptions *A4-A5-A6*, there exist $(z,t) \in Z^* \times \mathbb{R}_{++}$, such that $(0, t.z^+ + z) \in B''_i(p,q)$.$\square$

Assertion (ii) Let V be an open subset of $X^*_i \times Z^*_i$, such that $V \cap B''_i(p,q) \neq \varnothing$ and let $(x,z) \in V \cap B''_i(p,q)$. From the definition, there exists a neigbourhood, U, of (p,q), such that $(x,z) \in V \cap B''_i(p',q')$, if $(p',q') \in U$, i.e., B''_i is lower semicontinuous at (p,q).$\square$

We now introduce an agent representing markets $(i = 0)$, a convex compact set, $\Theta := P \times Q \times (\times_{i \in I} X^*_i \times Z^*_i)$, and a lower semicontinous reaction correspondence on the set Θ, for each agent. Thus, for every $i \in I$ and every $\theta := (p, q, [(x_i, z_i)]) \in \Theta$, we let:

$$\Psi_0(\theta) := \{\ (p', q') \in P \times Q : (q' - q) \cdot \sum_{i \in I} z_i + \sum_{s \in \underline{\mathbf{S}}'} (p'_s - p_s) \cdot \sum_{i \in I} (x_{is} - e_{is}) > 0\ \};$$

$$\Psi_i(\theta) := \begin{cases} B'_i(p,q) & if \quad (x_i, z_i) \notin B'_i(p,q) \\ B''_i(p,q) \cap P_i(x_i) \times Z^*_i & if \quad (x_i, z_i) \in B'_i(p,q) \end{cases}$$

Lemma 2 *For each* $i \in I \cup \{0\}$*,* Ψ_i *is lower semicontinuous.*

Proof See the Appendix. $\square$

The latter correspondences admit a fixed point, θ^*, along the following Claim:

Claim 4 *There exists* $\theta^* := (p^*, q^*, [(x_i^*, z_i^*)]) \in \Theta$*, such that*:

(i) $\forall (p,q) \in P \times Q,\ (q^* - q) \cdot \sum_{i \in I} z_i^* + \sum_{s \in \underline{\mathbf{S}}'} (p_s^* - p_s) \cdot \sum_{i \in I} (x_{is}^* - e_{is}) \geqslant 0$;

(ii) $\forall i \in I,\ (x_i^*, z_i^*) \in B_i'(p^*, q^*)$ *and* $B_i''(p^*, q^*) \cap P_i(x_i^*) \times Z_i^* = \varnothing$.

Proof Quoting Gale-Mas-Colell (1975, 1979): "*Given* $X = \times_{i=1}^m Xi$*, where* X_i *is a non-empty compact convex subset of* $\mathbb{R}^n$*, let* $\varphi_i : X \to X_i$ *be* m *convex (possibly empty) valued correspondences, which are lower semicontinuous. Then, there exists* $x := (x_i)$ *in* X *such that for each* i *either* $x_i \in \varphi_i(x)$ *or* $\varphi_i(x) = \varnothing$". The correspondences Ψ_i, for each $i \in I \cup \{0\}$, meet all conditions of the above theorem and yield Claim 4. □

3.3 An equilibrium of the economy $\mathcal{E}$

The above fixed point, θ^*, meets the following properties, proving Theorem 1:

Claim 5 *Given* $\theta^* := (p^*, q^*,\ [(x_i^*, z_i^*)]\ ) \in \Theta$*, along Claim 4, the following holds*:

(i) $\sum_{i \in I}\ z_i^{**} = 0$*, where, for each* $i \in I$*,* z_i^{**} *is the normal projection of* z_i^* *on* Z^**;*

(ii) $\sum_{i \in I} (x_{is}^* - e_{is}) = 0$*, for every* $s \in \underline{\mathbf{S}}'$*;*

(iii) for every $i \in I,\ (x_i^*, z_i^*) \in B_i'(p^*, q^*)$ *and* $B_i'(p^*, q^*) \cap P_i(x_i^*) \times Z_i^* = \varnothing$*;*

(iv) $p_s^* \in \mathbb{R}_{++}^{L \times \underline{\mathbf{S}}'}$ *and* $\gamma_{(p^*, q^*)} = 0$*;*

(v) $\sum_{i \in I}\ z_i^* \in Z^o$*, and we set as given* $(z_i^o) \in \times_{i \in I} Z_i^o$*, such that* $\sum_{i \in I}\ z_i^* = \sum_{i \in I}\ z_i^o$*;*

(vi) given $(z_i) := (z_i^* - z_i^o)$*,* $\overline{q} := \sum_{s \in \underline{\mathbf{S}}}\ \lambda_s V(s)$ *and* $q := (q^* + \overline{q}) \in \mathcal{NA}(\lambda)$*, the collection of prices and strategies,* $(p^*, q, [(x_i^*, z_i)])$*, defines an equilibrium of the economy* $\mathcal{E}$*.*

Proof Assertion (i) Assume, by contraposition, that $\sum_{i \in I}\ z_i^{**} \neq 0$. Then, from Claim 4-$(i)$, the relations $q^* \cdot \sum_{i \in I}\ z_i^{**} = q^* \cdot \sum_{i \in I}\ z_i^* > 0$ and $\gamma_{(p^*, q^*)} = 0$ hold. Moreover, from Claim 4-(i), the relations $0 \leqslant \sum_{i \in I}\ p_s^* \cdot\ (x_{is}^* - e_{is})$ hold, for every $s \in \underline{\mathbf{S}}'$. From Claim 4-$(ii)$, the relations $p_0^* \cdot\ (x_{i0}^* - e_{i0}) + q^* \cdot z_i^* + \sum_{s \in \underline{\mathbf{S}}}\ \lambda_s\ p_s^* \cdot (x_{is}^* - e_{is}) \leqslant 0$ hold, for every $i \in I$. Summing them up (for $i \in I$) yields, from above:

$$0 < p_0^* \cdot \sum_{i\in I} (x_{i0}^* - e_{i0}) + q^* \cdot \sum_{i\in I} z_i^* + \sum_{s\in \underline{\mathbf{S}}} \lambda_s \sum_{i\in I} p_s^* \cdot (x_{is}^* - e_{is}) \leqslant 0.$$

This contradiction proves that $\sum_{i\in I} z_i^{**} = 0$. □

Assertion (ii) From Claim 4-(i), $p_s^* \cdot \sum_{i\in I} (x_{is}^* - e_{is}) \geqslant 0$ holds, for every $s \in \underline{\mathbf{S}}'$, and $p_0^* \cdot \sum_{i\in I} (x_{i0}^* - e_{i0}) + \sum_{s\in \underline{\mathbf{S}}} \lambda_s \sum_{i\in I} p_s^* \cdot (x_{is}^* - e_{is}) > 0$ holds whenever $(\sum_{i\in I} (x_{is}^* - e_{is}))_{s\in \underline{\mathbf{S}}'} \neq 0$. Assume, by contraposition, that $\sum_{i\in I} (x_{is}^* - e_{is}) \neq 0$, for some $s \in \underline{\mathbf{S}}'$. Then, from Claim 4, the relation $\gamma_{(p^*,q^*)} = 0$ and the following budget constraints hold, for each $i \in I$:

$$p_0^* \cdot (x_{i0}^* - e_{i0}) + q^* \cdot z_i^* + \sum_{s\in \underline{\mathbf{S}}} \lambda_s p_s \cdot (x_{is} - e_{is}) \leqslant 0.$$

Summing them up yields, from Assertion (i) and above:

$$0 < p_0^* \cdot \sum_{i\in I} (x_{i0}^* - e_{i0}) + \sum_{s\in \underline{\mathbf{S}}} \lambda_s \sum_{i\in I} p_s^* \cdot (x_{is}^* - e_{is}) \leqslant 0.$$

This contradiction proves that $\sum_{i\in I} (x_{is}^* - e_{is}) = 0$, for every $s \in \underline{\mathbf{S}}'$. □

Assertion (iii) Let $i \in I$ be given. From Claim 4, it suffices to show the relation: $B_i'(p^*, q^*) \cap P_i(x_i^*) \times Z_i^* = \varnothing$. By contraposition, we assume that there exists a strategy, $(x_i, z_i) \in B_i'(p^*, q^*) \cap P_i(x_i^*) \times Z_i^*$, and we set as given $(x_i', z_i') \in B_i''(p^*, q^*) \subset B_i'(p^*, q^*)$, along Claim 3. From Assumptions *A3*-*A4*, the relations $(x_i^n, z_i^n) := [\frac{1}{n}(x_i', z_i') + (1 - \frac{1}{n})(x_i, z_i)] \in B_i''(p^*, q^*)$ hold, for every $n \in \mathbb{N}$, and $(x_i^N, z_i^N) \in P_i(x_i^*) \times Z_i^*$ holds, for $N \in \mathbb{N}$ big enough. The relation $(x_i^N, z_i^N) \in B_i''(p^*, q^*) \cap P_i(x_i^*) \times Z_i^*$ follows and contradicts Claim 4. □

Assertion (iv) From Assertion (ii), we may assume that $\|x_i^*\| \leqslant \sum_{i\in I} \|e_i\| < r$ holds for each $i \in I$ in Lemma 1. Then, the relation $p^* \in \mathbb{R}_{++}^{L\underline{\mathbf{S}}'}$ is standard from Assumption *A1* and Assertion (iii). It follows from Assumption *A1* and Assertion (iii) that the budget constraints of $(x_i^*, z_i^*) \in B_i'(p^*, q^*)$ are all binding. Summing them up, for $i \in I$, at the first period yields, from Assertions (i)-(ii):

$$0 = \sum_{i\in I} p_0^* \cdot (x_{i0}^* - e_{i0}) + q^* \cdot \sum_{i\in I} z_i^* + \sum_{s\in \underline{\mathbf{S}}} \lambda_s \sum_{i\in I} p_s^* \cdot (x_{is}^* - e_{is}) = \#I.\gamma_{(p^*,q^*)}.$$ □

Assertion (v) From Assertion (iii)-(iv) and Assumption $A1$, the budget constraints of $(x_i^*, z_i^*) \in B_i'(p^*, q^*)$ (for $i \in I$) are all binding. Then, the relations, $\sum_{i \in I} (x_{is}^* - e_{is}) = 0$, of Assertion (ii), yield: $0 = \sum_{i \in I} p_s^* \cdot (x_{is}^* - e_{is}) = \sum_{i \in I} V(s) \cdot z_i^* = V(s) \cdot \sum_{i \in I} z_i^*$, for each $s \in \underline{\mathbf{S}}$. The latter are written, with the notations of sub-Section 2.3, $(\sum_{i \in I} z_i^*) \in \underline{Z}^o$. Moreover, $\sum_{i \in I} z_i^{**} = 0$ holds from Assertion (i). It follows that $\sum_{i \in I} z_i^* \in Z^o$. □

Assertion (vi) Let $\mathcal{C} := (p^*, q, |(x_i^*, z_i)])$ be defined along Claim 5. That collection meets Conditions (b) and (c) of Definition 1, from Assertions (ii) and (v) above, and, moreover, $q \in \mathcal{NA}(\lambda)$ holds from the definition.

For each $i \in I$, the relations $(x_i^*, z_i) \in B_i(p^*, q)$ and $(x_i^*, z_i^*) \in B_i(p^*, q)$ hold from the definitions of z_i and q, and from Assertions (iii)-(iv)-(v) (which imply that all constraints of $(x_i^*, z_i^*) \in B_i'(p^*, q)$ are binding). The relation $\sum_{i \in I}(\|x_i^*\| + \|z_i^*\|) < r$ follows, from Lemma 1 and Assertions (ii)-(v), that is, (x_i^*, z_i^*) is interior to $X_i^* \times Z_i^*$.

Let $i \in I$ be given and assume, by contraposition, that $B_i(p^*, q) \cap P_i(x_i^*) \times Y_i \neq \varnothing$. Then, there exists $(x_i', z_i') \in B_i(p^*, q) \cap (P_i(x_i^*) \cap X_i^*) \times Z_i^*$, from the interiority of (x_i^*, z_i^*), Assumptions $A3$-$A4$-$A5$ and the definition of Z_i. Moreover, from Assumption $A1$ and Assertion (iv), we may assume that all budget constraints of (x_i', z_i') in $B_i(p^*, q)$ are binding. Then, the relation $(x_i', z_i') \in B_i'(p^*, q^*)$ holds from Assertion (iv) and the definition of $B_i'(p^*, q^*)$. It follows that $(x_i', z_i') \in B_i'(p^*, q^*) \cap P_i(x_i^*) \times Z_i^*$, which contradicts Assertion (iii). This contradiction ends the proof of Assertion (vi) and Theorem 1. □

3.4 The model's outcomes when one agent is fully informed

We now assume that one agent is fully informed and illustrate Theorem 1 simply.

Resuming the notations of sub-Section 2.3, the relation $Z^* = \{0\}$ (or $Z^o = \underline{Z}^o$) holds. The definition of no-arbitrage prices and the proof simplify, with $Q = \{0\}$:

Assumption *A5* is no longer required and may be smoothen to $Z_i^o + Y_i \subset Y_i$, for each $i \in I$, whose interpretation and relevance are clear. In Definition 2, the relations $\mathcal{NA}(\lambda) = \{\sum_{s \in \underline{\mathbf{S}}} \lambda_s V(s)\}$ hold, for every $\lambda := (\lambda_s) \in \mathbb{R}^{\underline{\mathbf{S}}}_{++}$, and the identity $\mathcal{NAC} = \mathcal{NA}$ obtains. A no-arbitrage price, $q \in \mathcal{NA}$, is characterized by the state prices of the informed agent, say $\lambda := (\lambda_s) \in \mathbb{R}^{\underline{\mathbf{S}}}_{++}$, and written as standard, $q = \sum_{s \in \underline{\mathbf{S}}} \lambda_s V(s)$.

With one fully informed agent, the two concepts of no-arbitrage price and equilibrium asset price coincide in a clear standard way from Theorem 1. This theorem extends Cass' to asymmetric information, restricted participation and non-ordered preferences. However, its proof drops the Cass trick, that is, defines budget sets symmetrically across agents. Similarly, Cornet-Gopalan (2010) drops the Cass trick, introduces restricted participation and deals with non-ordered preferences. But it ignores asymmetric information. The latter paper smoothens the above Assumption *A1* to a non-satiation condition, and drops the standard Assumption *A6* on the existence of a riskless portfolio. But it restricts participation conditions beyond Assumption *A4*. By another technique and up to the above changes, it proves a similar existence result as the current paper's for the symmetric information equilibrium.

De Boisdeffre (2021, a) also extends Cass' theorem to an asymmetric information setting, where one agent is fully informed. However, as Cass (1984), it builds on ordered preferences, unrestricted portfolio participation and uses the Cass trick. Moreover, it only yields interior equilibrium allocations, which are at odds with actual consumptions. The current model is not so restrictive. It is general enough to allow for any collection of information signals, intransitive preferences or border consumptions, and for a mild standard restriction on financial participation.

Appendix

Lemma 1 $\exists r > 0 : \forall (p,q) \in P \times Q,\ \forall\ [(x_i, z_i)] \in \mathcal{A}(p,q),\ \sum_{i\in I}(\|x_i\| + \|z_i\|)\ < r$

Proof Let $\delta = \sum_{i \in I} \|e_i\|$, $(p,q) \in P \times Q$ and $[(x_i, z_i)] \in \mathcal{A}(p,q)$ be given. The relations $x_{is} \in [0, \delta]^L$ hold, for every pair $(i,s) \in I \times \underline{\mathbf{S}}'$, from the market clearance conditions of $\mathcal{A}(p,q)$. From the fact that there exists $\alpha > 0$, such that $p_s^i \in [\alpha, +\infty[^L$, for every $(i,s) \in I \times S_i \backslash \underline{\mathbf{S}}$, it suffices to prove the following Assertion:

$\exists r' > 0,\ \forall (p,q) \in P \times Q,\ \forall\ [(x_i, z_i)] \in \mathcal{A}(p,q),\ \sum_{i\in I}\ \|z_i\|\ < r'$.

Assume, by contraposition, that, for every $k \in \mathbb{N}$, there exist $(p^k, q^k) \in P \times Q$ and $[(x_i^k, z_i^k)] \in \mathcal{A}(p^k, q^k)$, such that $\alpha_k := \sum_{i \in I}\ \|z_i^k\|\ > k$. For every $k \in \mathbb{N}$, we let $z^k := (z_i^k)$ and $z'^k := z^k/\alpha_k := (z_i^k/\alpha_k)$ belong to $\times_{i\in I} Z_i \cap Y_i$, from Assumption *A4*. The bounded sequence, $\{z'^k\}$, may be assumed to converge in a closed set, say to $z := (z_i) \in \times_{i \in I} Z_i \cap Y_i$, such that $\|z\| = 1$. The relations $[(x_i^k, z_i^k)] \in \mathcal{A}(p^k, q^k)$ hold, for every $k \in \mathbb{N}$, and imply, for every $(i,s,k) \in I \times S_i \times \mathbb{N}$:

$V(s) \cdot z_i^k \geqslant -\delta$, hence, $V(s) \cdot z_i'^k \geqslant -\delta/k$ and, in the limit, $V(s) \cdot z_i \geqslant 0$, for each $s \in S_i$;

$\sum_{i\in I}\ z_i'^k \in Z^o$, hence, $\sum_{i \in I}\ z_i \in Z^o$.

Let $\sum_{i\in I}\ z_i\ = \sum_{i\in I}\ z_i^o$, for some $(z_i^o) \in \times_{i\in I} Z_i^o$, be given. Then, the relations, $z_i^* := (z_i - z_i^o) \in Y_i$ and $V(s) \cdot z_i^* \geqslant 0$ hold, for each $(i,s) \in I \times S_i$, from above and from Assumption *A5*, whereas $\sum_{i\in I} z_i^* = 0$ holds. It follows from Cornet-De Boisdeffre (2002, p. 401) that $(z_i^*) \in \times_{i\in I} Z_i^o$, hence, that $z := (z_i) \in \times_{i \in I} Z_i^o \cap Z_i = \{0\}$. The latter relation contradicts the former, $\|z\| = 1$. This contradiction proves Lemma 1. □

Lemma 2 *For each* $i \in I \cup \{0\}$, Ψ_i *is lower semicontinuous.*

Proof The correspondence Ψ_0 is lower semicontinuous for having an open graph.

We now set as given $i \in I$ and $\theta := (p, q, [(x_i, z_i)]) \in \Theta := P \times Q \times (\times_{i\in I} X_i^* \times Z_i^*)$.

- Assume that $(x_i, z_i) \notin B_i'(p, q)$. Then, $\Psi_i(\theta) = B_i'(p, q)$.

Let V be an open set in $X_i^* \times Z_i^*$, such that $V \cap B_i'(p, q) \neq \varnothing$. It follows from the convexity of $B_i'(p, q)$ and the non-emptyness of the open set $B_i''(p, q) \subset B_i'(p, q)$ that $V \cap B_i''(p, q) \neq \varnothing$. From Claim 3, there exists a neighborhood U of (p, q), such that $V \cap B_i'(p', q') \supset V \cap B_i''(p', q') \neq \varnothing$, for every $(p', q') \in U$.

Since $B_i'(p, q)$ is nonempty, closed, convex in the compact set $X_i^* \times Z_i^*$, there exist open sets V_1 and V_2 in $X_i^* \times Z_i^*$, such that $(x_i, z_i) \in V_1$, $B_i'(p, q) \subset V_2$ and $V_1 \cap V_2 = \varnothing$. From Claim 2, there exists a neighborhood $U_1 \subset U$ of (p, q), such that $B_i'(p', q') \subset V_2$, for every $(p', q') \in U_1$. Let $W = U_1 \times (\times_{j\in I} W_j)$, where $W_i := V_1$, and $W_j := X_j^* \times Z_j^*$, for each $j \in I \backslash \{i\}$, be a neighbourhood of θ in Θ. Then, $\Psi_i(\theta') = B_i'(p', q')$, and $V \cap \Psi_i(\theta') \neq \varnothing$ hold, for every $\theta' := (p', q', [(x_i', z_i')]) \in W$. That is, Ψ_i is lower semicontinuous at θ. $\square$

- Assume that $(x_i, z_i) \in B_i'(p, q)$. Then, $\Psi_i(\theta) = B_i''(p, q) \cap P_i(x) \times Z_i^*$.

Lower semicontinuity results from the definition if $\Psi_i(\theta) = \varnothing$. Assume, now, that $\Psi_i(\theta) \neq \varnothing$. We notice that P_i is lower semicontinuous with open values, from Assumption *A3*, and that B_i'' has an open graph in $P \times Q \times X_i^* \times Z_i^*$. As a corollary, the correspondence $(p', q'[(x_i', z_i')]) \in \Theta \mapsto B_i''(p', q') \cap P_i(x_i') \times Z_i^* \subset B_i'(p', q')$ is lower semicontinuous at θ, and Ψ_i is also, from the latter inclusions. $\square$

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